$x=3,\ y=-3,\ z=-7$

$r^2=x^2+y^2 \Rightarrow r^2=3^2+3^2=18 \Rightarrow r=3\sqrt{2}$

$\tan\theta=\frac{y}{x} \Rightarrow \tan\theta=\frac{-3}{3}=-1 \Rightarrow \theta=\frac{7\pi}{4}+2k\pi$

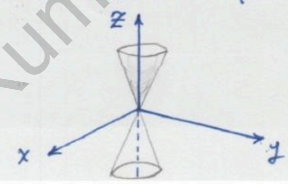
$z=-7 \quad \Rightarrow \left(3\sqrt{2}, \frac{7\pi}{4}, -7\right)$ is the coordinates of the point in cylindrical coordinate system.

Note: cylindrical coordinates are useful in problems that involve symmetry about an axis, and the z-axis is chosen to coincide with this axis of symmetry.

Example: Describe the surface whose equation in cylindrical coordinates is $z=r$.

$$\left.\begin{array}{l} z=r \Rightarrow z^2=r^2 \\ r^2=x^2+y^2 \end{array}\right\} \Rightarrow z^2=x^2+y^2$$

and we know that this is the equation of a circular cone whose axis is the z-axis



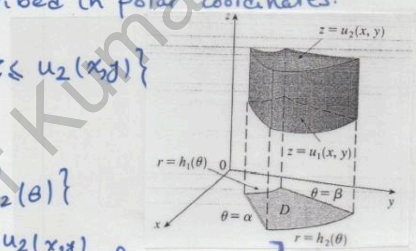
Evaluating Triple Integrals w/ Cylindrical Coordinates:

If E is a region (suppose that it is a type I region) whose projection D onto the xy-plane is conveniently described in polar coordinates:

$$E=\{(x,y,z) \mid (x,y)\in D,\ u_1(x,y)\leq z\leq u_2(x,y)\}$$

& D is given in polar coordinates by

$$D=\{(r,\theta) \mid \alpha\leq\theta\leq\beta,\ h_1(\theta)\leq r\leq h_2(\theta)\}$$



we know that $\iiint_E f(x,y,z)\,dV=\iint_D\left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\right]dA$

and we know how to evaluate double integrals in polar coordinates, we can rewrite the above equation as:

$$\boxed{\iiint_E f(x,y,z)\,dV=\int_\alpha^\beta\int_{h_1(\theta)}^{h_2(\theta)}\int_{u_1(r\cos\theta,\, r\sin\theta)}^{u_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta}$$

Note: The above equation says that to convert a triple integral from rectangular to cylindrical coordinates, we should write $x=r\cos\theta$, $y=r\sin\theta$, leaving z as it is, using appropriate limits of integration for z, r, and θ and replacing dV by $r\,dz\,dr\,d\theta$.

It is worthwhile to use cylindrical coordinates, especially when $f(x,y,z)$ involves x^2+y^2 expression.

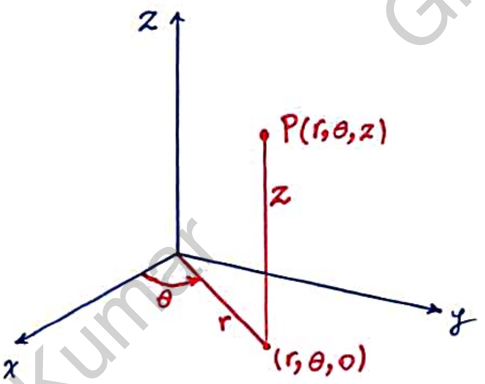
Introduction to Cylindrical Coordinates:

What was the reason that sometimes in plane geometry, we used polar coordinates instead of Cartesian Coordinates? The reason was that for some problems, Polar coordinates gave us a more convenient description of certain curves/Regions.

In three-dimensions (Not plane geometry or 2D), we define a new coordinate system called cylindrical coordinates, that is similar to Polar Coordinates except that it has an additional element.

Cylindrical Coordinates:

In the cylindrical coordinate system, a point P in three-dimensional space is represented by the ordered triple (r, θ, z), where r, and θ are polar coordinates of the projection of P onto xy-plane and z is the direct distance from xy-plane to P.



To convert from rectangular to cylindrical coordinates, and vice versa, we use:

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$
$r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$, $z = z$

Example: plot the point with cylindrical coordinates $(2, 2\pi/3, 1)$ and find its rectangular coordinates.
Find cylindrical coordinates of the point w/ rectangular coordinates $(3, -3, -7)$.

Solution: we know $r = 2$, $\theta = \frac{2\pi}{3}$, and $z = 1$

$$\Rightarrow \left.\begin{array}{l} x = r\cos\theta = 2\cdot\cos\frac{2\pi}{3} = 2(-\tfrac{1}{2}) = -1 \\ y = r\sin\theta = 2\cdot\sin\frac{2\pi}{3} = 2(\frac{\sqrt{3}}{2}) = \sqrt{3} \\ z = 1 \end{array}\right\}$$

$(-1, \sqrt{3}, 1)$ is the coordinates of the point in rectangular coordinate system